Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9415179200

FURTHER MATHEMATICS **9231/22**

Paper 2 **October/November 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.

JC18 11_9231_22/RP

CAMBRIDGE International Examinations

[Turn over

1 The point O is on the fixed horizontal line l. Points A and B on l are such that $OA = 0.1$ m and $OB = 0.5$ m, with A between O and B. A particle P oscillates on l in simple harmonic motion with centre O. The kinetic energy of P when it is at A is twice its kinetic energy when it is at B. Find the amplitude of the motion. [3]

....

....

....

....

....

....

....

....

....

....

....

2 Two uniform small smooth spheres A and B have equal radii and masses $2m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$.

(i) Find, in terms of u, the speeds of A and B after this collision. [4]

....

....

....

....

....

....

....

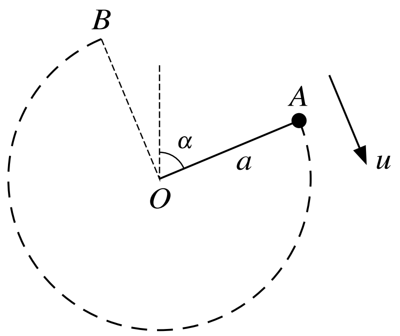
....

....

Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $\frac{1}{2}$.

(ii) Find, in terms of d and u, the time that elapses between the first and second collisions between A and B. [5]

3



A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The point A is such that $OA = a$ and OA makes an angle α with the upward vertical, where $\tan\alpha = \frac{12}{5}$. The particle is projected downwards from A with speed u perpendicular to the string and moves in a vertical plane (see diagram). The string becomes slack after the string has rotated through 270° from its initial position, with the particle now at the point B.

(i) Show that $u^2 = 2ag$. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

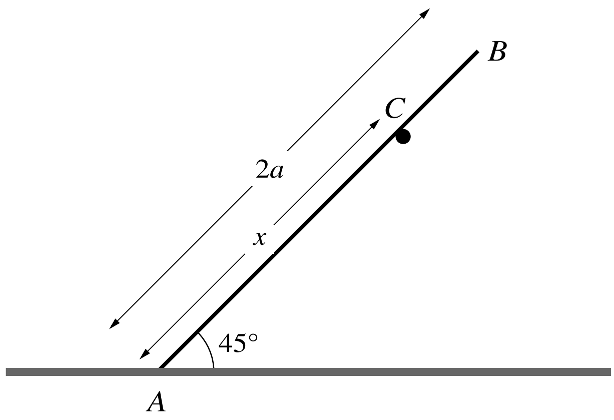
....

....

....

(ii) Find the maximum tension in the string as the particle moves from A to B. [4]

4



A uniform rod AB of length $2a$ and weight W rests against a smooth horizontal peg at a point C on the rod, where $AC = x$. The lower end A of the rod rests on rough horizontal ground. The rod is in equilibrium inclined at an angle of 45° to the horizontal (see diagram). The coefficient of friction between the rod and the ground is μ. The rod is about to slip at A.

(i) Find an expression for x in terms of a and μ. [5]

(ii) Hence show that $\mu \geqslant \frac{1}{3}$. [2]

(iii) Given that $x = \frac{3}{2}a$, find the value of μ and the magnitude of the resultant force on the rod at A. [4]

5 An object is formed from a uniform circular disc, of radius $2a$ and mass $3M$, and a uniform rod AB, of length $3a$ and mass kM, where k is a constant. The centre of the disc is O. The end B of the rod is rigidly joined to a point on the circumference of the disc so that OBA is a straight line. The fixed horizontal axis l is in the plane of the object, passes through A and is perpendicular to AB.

(i) Show that the moment of inertia of the object about the axis l is $3Ma^2(26 + k)$. [5]

The object is free to rotate about l.

(ii) Show that small oscillations of the object about l are approximately simple harmonic. Given that the period of these oscillations is $4\pi\sqrt{\left(\frac{a}{g}\right)}$, find the value of k. [6]

6 The heights, in metres, of a random sample of 8 trees of a particular type are as follows.

14.2	11.3	10.8	8.4	12.8	11.5	12.1	9.2

Assuming that heights of trees of this type are normally distributed, calculate a 95% confidence interval for the mean height of trees of this type. [6]

7 The continuous random variable X has distribution function given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{90}(x^2 + x^4) & 0 \leqslant x \leqslant 3, \\ 1 & x > 3. \end{cases}$$

The random variable Y is defined by $Y = X^2$.

(i) Find the probability density function of Y. [4]

(ii) Find the mean value of Y. [2]

8 Lan starts a new job on Monday. He will catch the bus to work every day from Monday to Friday inclusive. The probability that he will get a seat on the bus has the constant value p. The random variable X denotes the number of days that Lan will catch the bus until he is able to get a seat. The probability that Lan will not get a seat on the Monday, Tuesday, Wednesday or Thursday of his first week is 0.4096.

(i) Show that $p = 0.2$. [2]

(ii) Find the probability that Lan first gets a seat on Monday of the second week in his new job. [2]

(iii) Find the least integer N such that $P(X \leqslant N) > 0.9$, and identify the day and the week that corresponds to this value of N. [4]

9 For a random sample of 5 observations of pairs of values (x, y), the equation of the regression line of y on x is $y = 4.2 + cx$ and the equation of the regression line of x on y is $x = 10.8 + dy$, where c and d are constants. The product moment correlation coefficient is -0.7214 and the mean value of x is 7.018.

(i) Test at the 5% significance level whether there is evidence of non-zero correlation between the variables. [4]

....

....

....

....

....

....

....

....

....

....

(ii) Find the values of c and d. [5]

....

....

....

....

....

....

....

....

....

....

....

(iii) Use an appropriate regression line to estimate the value of x when $y = 3.5$, and comment on the reliability of your estimate. [2]

10 The number of accidents, x, that occur each day on a motorway are recorded over a period of 40 days. The results are shown in the following table.

Number of accidents	0	1	2	3	4	5	6	⩾7
Observed frequency	3	5	8	10	5	7	2	0

(i) Show that the mean number of accidents each day is 2.95 and calculate the variance for this sample. Explain why these values suggest that a Poisson distribution might fit the data. [3]

....

....

....

....

....

....

....

....

A Poisson distribution with mean 2.95, as found from the data, is used to calculate the expected frequencies, correct to 2 decimal places. The results are shown in the following table.

Number of accidents	0	1	2	3	4	5	6	⩾7
Observed frequency	3	5	8	10	5	7	2	0
Expected frequency	2.09	6.18	9.11	8.96	6.61	3.90	1.92	1.23

(ii) Show how the expected frequency of 6.61 for $x = 4$ is obtained. [2]

....

....

....

....

....

....

....

....

(iii) Test at the 5% significance level the goodness of fit of this Poisson distribution to the data. [7]

11 Answer only **one** of the following two alternatives.

EITHER

One end of a light elastic spring, of natural length 0.8 m and modulus of elasticity 40 N, is attached to a fixed point O. The spring hangs vertically, at rest, with particles of masses 2 kg and M kg attached to its free end. The M kg particle becomes detached from the spring, and as a result the 2 kg particle begins to move upwards.

(i) Show that the 2 kg particle performs simple harmonic motion about its equilibrium position with period $\frac{2}{5}\pi$ s. State the distance below O of the centre of the oscillations. [7]

The speed of the 2 kg particle is $0.4\,\text{m s}^{-1}$ when its displacement from the centre of oscillation is 0.06 m.

(ii) Find the amplitude of the motion. [3]

(iii) Deduce the value of M. [4]

OR

In a particular country, large numbers of ducks live on lakes A and B. The mass, in kg, of a duck on lake A is denoted by x and the mass, in kg, of a duck on lake B is denoted by y. A random sample of 8 ducks is taken from lake A and a random sample of 10 ducks is taken from lake B. Their masses are summarised as follows.

$$\Sigma x = 10.56 \qquad \Sigma x^2 = 14.1775 \qquad \Sigma y = 12.39 \qquad \Sigma y^2 = 15.894$$

A scientist claims that ducks on lake A are heavier on average than ducks on lake B.

(i) Test, at the 10% significance level, whether the scientist's claim is justified. You should assume that both distributions are normal and that their variances are equal. [9]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

A second random sample of 8 ducks is taken from lake A and their masses are summarised as

$$\Sigma x = 10.24 \qquad \text{and} \qquad \Sigma(x - \bar{x})^2 = 0.294,$$

where $\bar{x}$ is the sample mean. The scientist now claims that the population mean mass of ducks on lake A is greater than p kg. A test of this claim is carried out at the 10% significance level, using only this second sample from lake A. This test supports the scientist's claim.

(ii) Find the greatest possible value of p. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE